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The Theory Behind Economic Values

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THE THEORY BEHIND ECONOMIC VALUES

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1. Introduction

The Purpose of the Paper

This paper provides a nontechnical description of the theoretical underpinnings and meanings within economics of notions of value. The analysis presented here provides a guide to the way economists measure the value of goods and services. It is not required that these goods and services be directly bought and sold in markets. In the case of the fishery, for example, this means that the discussion is relevant to the valuation of both commercial and recreational goods. The commercial fishery provides a product (harvested fish) that is bought and sold (marketed). In a recreational setting, anglers usually do not directly purchase fish. They may, however, pay to gain access to a fishery (site). Some individuals may even be willing to pay for the continued existence of fish stocks even though they don't harvest them.

The work in this paper is guided, in part, by the observation that, despite the vast literature that has accumulated on economic valuation, considerable confusion continues to surround the notions of value used by economists. Economists have long debated theoretical issues in valuation and many of the problems have now been resolved. Nonetheless, misconceptions and debates about appropriate interpretations and applications still arise. In large measure this seems to reflect important misunderstandings about the underpinnings of the economic theory of value and about the precise nature of conceptual and empirical (statistical) experiments in economics. Perhaps more so than ever before, it is easy to become confused by discussions of value and valuation in economics. The field is fraught with subtle but important technical details. Ultimately, our interest must focus on the practical application of economic values. One of the most important aspects of this exposition is that it provides some insight about how an economist reasons in attempting to solve a valuation problem. It is hoped that the discussion of economic reasoning contained in this paper will help to explain with more precision what the valuation numbers produced by economists mean and how they can be used for policy analysis. The paper is not intended, however, as a manual on how to calculate economic values in an applied setting.

The approach adopted in this paper differs somewhat from the standard way economists have tended to explain their models of valuation and subsequently present their value estimates. Rather than start the discussion at the relatively advanced stage of demand curves,[1] we hope to sidestep many of the standard problems of exposition that can arise by instead beginning with a largely nontechnical discussion of the three notions that provide the logical foundation of the economic theory of value. In particular, we introduce, discuss and join the notions of: (1) preferences; (2) budgets; and (3) rationality. When attempting to resolve issues in valuation, economists often work at the level of these basic notions. Our approach leads logically to a variety of demand curves that often appear in existing (standard) approaches to explaining economic values and valuation. These demand curves can sometimes provide a visual representation of different value measures. One benefit of the approach adopted here is that a basis is provided for carefully explaining how

[1]A demand curve for a good by an individual shows how the individual will trade off price per unit and quantity when purchasing the good. It can be visualized as a graph of price against quantity. For a given price per unit, the quantity component is the number of units of the good that the individual will <u>definitely</u> buy at that price.

demand curves arise in economic theory and how they may provide a tool for understanding valuation problems.

As noted above, this paper presents a nontechnical discussion and explanation of economic valuation. We have attempted to limit the use of mathematics to some summary notation and to working out some numerical examples. Similarly, diagrams are introduced only when it is felt they may significantly help in the presentation of an idea or link our discussion to what has been presented by others. We again stress that this paper is not intended as a manual of rules of thumb for calculating economic values in practical settings. Rather, the intent is to focus on the conceptual issues of how values for goods and services emerge from economic theory, what they mean and, when properly measured, how they can be used in practice.

The Presentation of the Paper

We begin in Section 2 by examining a list of illustrative questions that are often asked in valuation settings. An example of one such question is: 'What is the value of a particular recreational fishery?' We do not immediately suggest answers to the questions or illustrate how the answers can be used to determine whether a particular policy should be initiated, continued or ended. Rather, we attempt to show that considerable confusion and ambiguity can arise with the very meaning of basic questions posed in economic valuation exercises. We suggest that there can reasonably be many different meanings associated with the term 'value'. As well, we point out that some common questions may really be too vague to be answered by a unique economic value. Later in the paper we return to these questions and explain how this ambiguity is handled in economic theory.

In Section 3 we present the fundamental building blocks of the economic theory of value. It is standard in economics to assume that the needs and desires of a consumer can be described by what is termed a 'preference relation'. In a sense that will be made explicit, the preference relation is a technical description of what allows the consumer to rank different combinations of goods and services from best to worst. The consumer is further assumed to be constrained in his or her desired purchases by a budget. Not all consumption patterns that a consumer is logically able to rank are also affordable. Finally, the individual is assumed to act rationally: he or she always chooses the best-ranked affordable combination of goods and services. An individual who is consuming his best-ranked affordable bundle is said to be 'in equilibrium'. The consumer equilibrium will tend to change as elements of the budget (prices, income, limited availability of some goods and services) change. An important aspect of the analysis will be to link the wellbeing of a consumer to the equilibrium he occupies. This link allows us to show how economists use the foregoing theoretical structure to define values in both a 'total' and an 'incremental' sense. Total value refers to the value placed on a given number of units of a good while incremental value refers to the value associated with one more unit of a good.

The material in Section 3 is presented at an abstract but intuitive level. In contrast, Section 4 provides an extensive example of how the theory is implemented in a variety of ideal settings. This example accomplishes three goals. First, it illustrates how a specific preference relation, a budget constraint and the rationality postulate lead to a consumer equilibrium. Secondly, we can use the example to re-explore the possible meanings of the valuation questions raised in Section 2. Finally, the example provides a natural way to consider one notion of a demand curve and to relate it to economic values. Of course, one example cannot illustrate or explain all of the theoretical and practical details that arise in valuation. It can nonetheless help to clarify the reasoning and types of answers that an economist will provide to a variety of valuation questions.

This paper attempts to cover a considerable amount of economic ground in a brief but thorough fashion. It is not an 'easy read' simply because the topics covered cannot be made 'easy'. Economic values lie at the heart of a very complex branch of economic theory. To understand these notions of value and how they can be used in practice is to understand a considerable amount of difficult economic theory.

2. *Valuation Questions: A First Look*

Some Common Questions

The questions posed in any valuation exercise in part reflect the special circumstances of the resource or good being examined. The following types of questions may reasonably arise during the valuation of a recreational or commercial fishery. Both of these fisheries may share the same physical location such as a lake.

1. What is the value of the commercial fishery?
2. What is the value of the recreational fishery?
3. Suppose there are separate quotas for the commercial and the recreational fishery. If the total quota is to be increased by one fish, should this fish be assigned to the recreational or the commercial fishery?
4. Would there be a net benefit from changing the division of a total quota between recreational and commercial fishing? That is, is there a benefit to increasing the size of one fishery and reducing the other by an equal amount?

5. How would the value of the recreational fishery change if its quality was improved by, say, reducing predators (e.g. lamprey), reducing congestion by restricting access or improving the habitat by reducing pollution?
6. What is the value of a fishing trip to an individual? What is the value of the next trip?

Economists provide answers to these questions based on the economic theory of value. While it is probably true that few individuals would disagree about the gist of the broad ideas present in any one of the above questions, all of the questions are ambiguous in several important ways. They will generally be understood by non-economists in ways that are much different than the way the economist undertaking the analysis understands them. In what follows we will investigate two sources of ambiguity. The first concerns the term 'value'. The standard economic model of consumer behaviour may lead to more than notion of value. Worse still for expository purposes, not all of these notions are measured in the same units for all individuals and are thus not comparable. Some common ground must be found.

The second source of ambiguity is more subtle but may be equally as important. The value estimate that an economic model provides in response to any of the foregoing questions depends upon how the question is posed within the model. In the form they are presented above, the six questions can each be introduced into a model in a variety of ways. Answers will be sensitive to how the question is posed.

Several Meanings of the Term 'Value'

The problem of what is meant by value and how it should be measured is recurrent in economics and certainly complicates resource valuation. There are at least four broad senses to the term value as it appears in, for example, question 1: "What is the value of the commercial fishery?" We consider each in turn. It is important to keep in mind that the issues raised here will be taken up in much more detail when the economic model of valuation is introduced in Section 3. The reader may begin to wonder why so many details must be addressed and why so many distinctions must be made. The major problem is that words and phrases that may have one meaning in standard usage outside of an economic model have yet another meaning within an economic model. It is useful to point in advance to where some of these problems may be encountered. At this stage the reader will no doubt have his own understanding of what economic values mean. This understanding will likely coincide with one of the four value meanings introduced below. A two-part test of the success of the presentation in this paper is that, after the paper has been read the reader (1) will understand the valuation approach proposed by economists and (2) will see why other notions are not consistent with the abstract general model adopted by economists.

PSYCHIC VALUE

a. First, value in question 1 ("What is the value of the commercial fishery?) could refer to the change in psychic or psychological wellbeing that would result from removing the commercial fishery.[2] Value, so defined, would be measured in abstract and generally unobservable units (of wellbeing). Further, there is no reason to suppose that the wellbeing scale is the same for each individual.[3] As such, it will not be possible to accumulate these units to measure total wellbeing or value of everyone. Economic theory

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[2]Here we measure the value of a good by comparing total value with the good and then with the good taken away.

[3]If people are not identical it becomes very difficult to judge who is better off using strictly objective criteria. For example, which of two individuals is better off if each would prefer to consume the bundle of goods and services that the other has purchased?

almost never assumes that all individuals are strictly identical in terms of psychological units of wellbeing. From a theory perspective, this would be unnecessarily restrictive. As a result, economic valuation studies do not attempt to measure levels of psychological wellbeing.

PSYCHIC DOLLAR EQUIVALENTS

b. Second, value could refer to the dollar equivalent of the psychic wellbeing noted in (a). Unfortunately, even in an ideal setting with perfect information about the budget and purchasing activities of a consumer, it is not possible to construct such a dollar equivalent. Under extremely unlikely conditions a dollar measure can be found that is proportional to changes in psychic value. The factor of proportionality can never be observed and will in general differ from person to person. As we will note later in this paper, a general theory of value does not require a measurable or observable relationship between changes in units of psychic wellbeing and dollars. As such, valuation studies in economics do not attempt to measure this second notion of value.

TOTAL PAYMENTS

c. A third measure of the value of a good might be the total payment that the individual makes for the good. In a market setting where prices of goods and services are determined by the forces of supply and demand, total expenditures on a good will often provide a poor indication of the importance of the good to the consumer. One reason lies in the fact that market price reflects both conditions of supply as well as demand. For example, water is, in some ways, invaluable but individuals usually pay much less per unit of water than per unit of diamonds. The low market price for water often reflects more the fact that it is plentiful than the fact that it is essential for life.

COMPEN-SATION d. A fourth way that value could be interpreted in question 1 ("What is the value of the commercial fishery?") is in terms of the dollar payment that a consumer would make (needs to receive) in order to keep psychic wellbeing fixed at some level. For example, suppose that you benefit from being able to consume fish caught by the commercial fishery and that 'removing part of the commercial fishery' would tend to reduce your psychic wellbeing. We could 'remove part of the commercial fishery' by placing a maximum on the total amount of commercial fishery output you could consume and making sure that this maximum amount is less than the amount you would freely choose. In contrast, receiving a gift of some money would tend to increase your wellbeing. Now consider the following (abstract) balancing exercise. More and more of the commercial fishery is removed (the maximum amount that you can consume of the fishery products is reduced) and, at the same time as each subsequent bit of the fishery is taken away, you are compensated with just enough income so that you agree that you are just as well off as before the bit was removed. Eventually your access to the fishery will have been removed entirely (you are not allowed to consume any fishery products) and you will have a sum of increases to your income. In a sense, the sum of the payments to you could be considered the value of the commercial fishery to you.[4]

It is important to note that this last measure of value does not try to create a direct link between units of (psychic) wellbeing and money. It therefore differs from the value measure considered in (b). In (d) we are interested in the smallest income payment needed to keep wellbeing unchanged regardless of whether wellbeing can be measured in some units. In fact, the interesting measures of economic value are generated using this and related approaches to compensation.

The distinctions between the above notions of value are very important within an economic setting. It is unlikely that the foregoing discussion will have illustrated these distinctions except in a vague sense. A more complete description of the process of economic reasoning and modelling must be presented in order to clarify the relationships between these various notions of value. This is the goal of the discussion presented in Section 3.

Finally, it is worth noting that it is the compensation notion of value appearing in (d) that economists tend to use. These values can be computed for an individual given that there is sufficient observational data about his or her purchasing behaviour. In computing these measures there is no need to quantify psychic wellbeing. Alternatively, the (d) values can be determined, in principle, from (truthful) answers to questions directed to the individual in an interview or as part of a questionnaire or survey. We take up these applied issues in the companion paper on measurement.

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[4]With only slight modification, this general approach could be used to find a value for the recreational fishery (Question 2).

3. Some Economic Theory of Value

An Overview of the Analysis

This section introduces the fundamental components of the economic theory of value. One of the important questions that the theory must address is the following: Why is a given individual observed to purchase a given collection of goods and services? In a sense, by demanding different goods and services, consumers make these goods valuable.[5] In order to understand how value can be measured within an economic setting it is first necessary to understand the decision making behaviour of consumers and, in particular, what leads them to purchase given combinations of goods and services.

The approach adopted by economists to model the choices of consumers is based, in part, on the conjecture that the observed behaviour of a consumer (i.e. his purchase of a given collection of goods and services) represents the 'best' decision of that consumer given needs and wants and any constraints (financial or otherwise) placed on his behaviour. Observed behaviour, then, is thought of as the 'equilibrium' outcome of a possibly very complex decision making process. During this process a 'best' combination of goods and services is determined.

The process which determines the equilibrium behaviour of the individual is examined in three parts. The first involves the needs and wants of the consumer. These are described by the consumer's preferences. The second part of the analysis involves describing the constraints faced by the consumer. The standard constraint requires that total expenditure on all goods and services may not exceed available income at any point in time. The first two parts of the model are then joined by what is referred to as the rationality assumption: The consumer chooses from the

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[5]If something is unwanted by all, it has no value in an economic setting.

affordable collections of goods and services that bundle which is most preferred. The consumer equilibrium is just that bundle.

In the subsections that follow we examine the three components of the model in detail and show how an equilibrium arises. We then point out how the model provides a variety of measures of value. Finally, we show how, in principle, values can be affected by changes in tastes and changes in the financial and other constraints faced by the individual.

The Consumer's Preferences

This section follows a long and slightly complicated route. The ultimate goal is straightforward: we want to find a simple and logically consistent theory of how consumers would choose between any two bundles of goods and services. We hope that the theory will be rich enough to include notions of best and worst bundles of goods and services and will ultimately provide a foundation for valuation. Unfortunately, many details have to be examined along the way.

In economic models individuals are treated impersonally and perhaps somewhat like automata. In the initial stages of analysis, most of what economists need to know about consumers is that they are able to rank different combinations (bundles) of goods and services. For example, bundle *A* may contain 4 pounds of food, 2 opera tickets and 1 fishing trip lasting one day at a particular site and bundle *B* may contain 4 pounds of food, 1 opera ticket and 3 fishing trips. It is assumed that the individual can name the bundle (*A* or *B*) that he prefers. As well, it is assumed that this comparison can be extended to all possible combinations of goods and services. The technical term for this ranking mechanism is the 'preference relation' of the consumer. It is important to note that economists do not require any psychological information about why the ranking was reached. As well, it is typically assumed that the ranking provided by a consumer is independent of the market prices of the goods and services. What matters in being able to rank a set of bundles is the quantities of goods and services in the bundles and not whether the bundles are affordable or, in fact, ever purchased.[6]

The formal analysis of the consumer can be simplified somewhat by introducing stronger assumptions about the nature of the preference relation. Often it is assumed that when presented with a fixed group of bundles (no matter how many bundles in the group nor how close any two bundles may be in terms of quantities of goods contained) the consumer is always able to identify a 'best' bundle in the sense that no other bundle is preferred. Two or more bundles may tie for best. The consumer would then be indifferent between these bundles. Finally, it is again useful to point to the assumption that the ability to rank a group of bundles has nothing to do with being able to afford some or any of the bundles.

An important feature of the preference relation described above is that it can often be 'represented' by a numerical function defined over bundles of goods and services. This function is referred to as a utility function. Because the term utility arises so frequently in valuation theory, it is important to examine the precise sense in which a utility function can represent the preferences of a consumer. Before turning to this, it is useful to first introduce some notation and provide a definition of the term function that will be useful for our purposes.

Some Notation

We suppose that there are several different goods and services over which the consumer has preferences. Let there be 'n' such goods and services. For example, a case of 3 goods and services

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[6]This is not unreasonable. A individual may rank two expensive sports cars on the basis of colour or performance characteristics even though he or she is not able to afford either car.

($n = 3$) would arise in a world where there was only food (measured in pounds of beef), opera tickets and fishing trips (measured in days spent at the only available site).

Let x denote a generic bundle of goods and services with x_1 units of good 1, x_2 units of good 2 and so on up to x_n units of good n. We can denote a specific bundle (for example, bundle A) with a superscript on x. For example, x^A is bundle A and it contains x_1^A units of good 1, x_2^A units of good 2 and so on up to x_n^A units of good n. A compact way of displaying this information is to write $x = (x_1, x_2, \ldots, x_n)$ for the generic bundle and $x^A = (x_1^A, x_2^A, \ldots, x_n^A)$ for the specific bundle, A. The brackets enclose the component goods and services of the bundles.

Continuing the three good example from above, the generic bundle would be $x = (x_1, x_2, x_3)$ and a specific bundle (say, A) might be written: $x^A = (4, 2, 1)$. Bundle A contains 4 pounds of beef, 2 opera tickets and 1 fishing trip.

Functions

For our purposes it is sufficient to think of a function as a 'black box' with a meter on the outside. A bundle (for example, x^A) is put inside the black box and a number associated with the bundle is displayed on the meter. The number is unique in the sense that if you took out the bundle and then put it back in again, you would see the same meter reading. Different functions can be thought of as different black boxes. In general, when a given bundle is placed in a different black box, a different meter reading is also observed. Indeed, the units of the meter may be different and perhaps not even comparable to those of the original function (black box).

Suppose we denote a given function by the letter U. We sometimes make explicit the fact that the value of the function (meter reading) depends upon the bundle being evaluated. The notation for this is: $U(x)$ or $U(x_1, x_2, \ldots, x_n)$ for the generic bundle and $U(x^A)$ or $U(x_1^A, x_2^A, \ldots, x_n^A)$ for the specific bundle A. In terms of our numerical example with three goods, the function value or meter reading is $U(4, 2, 1)$.

Exactly how does the black box or function operate so as to associate a unique meter reading with a given bundle? We note at the outset that while one function may evaluate a bundle in a given way there is always another that will yield a different result. The number of ways that a bundle can be evaluated is infinite. The following functions have been chosen to illustrate three possibilities. The functions are distinguished by a superscript number: 1, 2, or 3.

1. $U^1(x) = x_1 + x_2 + \cdots + x_n$
2. $U^2(x) = \sqrt{x_1} + \sqrt{x_2} + \cdots + \sqrt{x_n}$
3. $U^3(x) = x_1 \cdot x_2 \cdots x_n$

Function 1 generates the meter reading by summing the quantities of the different goods and services in the bundle. Alternatively, function 2 calculates the sum of the square roots of the quantities of the components of the bundle. Finally, function 3 gives the product of the quantities in the bundle.

We can apply these functions to the example bundle $x^A = (4, 2, 1)$. The following results are obtained:

1. $U^1(4, 2, 1) = 4 + 2 + 1 \qquad = 7$
2. $U^2(4, 2, 1) = \sqrt{4} + \sqrt{2} + \sqrt{1} \qquad = 4.4142$
3. $U^3(4, 2, 1) = 4 \cdot 2 \cdot 1 \qquad = 8$

In all three cases the value of the function (meter reading) is different. There is no special significance to this. This result occurs because different functions will generally associate different values to a given bundle. Furthermore, the units of the functions are not directly comparable. For example, function 1 is measured in terms of the sum of pounds of beef plus opera tickets plus days at a site whereas function 3 is in units of the product of the quantities of the goods. There is no straightforward way to compare the units from these two functions. That is, it is not helpful to note that function 3 gives a reading that is one unit greater than function 1 (8 units versus 7 units) because the units are not comparable. Note however that we could reasonably compare the meter readings for a group of bundles if they were all evaluated using the same function. In that case, the function values would all be in the same units and hence comparable.

Utility Functions of Consumers

Earlier in the section we introduced the individual's preference relation whereby he or she could rank a collection of bundles from best to worst and we allowed ties in the rankings. At the time we were vague about the actual mechanics of how the ranking was generated. The preference relation did it all and that was all we needed to know. On a related note, throughout this paper we have often referred to the wellbeing of an individual. We have talked about the level of wellbeing of a consumer as if this level referred to a number. At this point we can better explain the reasons for our exposition. In particular, we will examine how a function can 'represent' the preferences of an individual and thus supply more insight into the process by which individuals rank bundles of goods and services.

Consider an individual whose preference relation allows him to rank any collection of bundles in the way we have discussed. A recent theorem in economic theory asserts that with only a few restrictions[7] on the nature of that preference relation, there will exist a function that provides an identical ranking to the one given by the preference relation on a bundle to bundle basis. This is a fundamental result and it is important to see how it works. Recall that a function uniquely associates

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[7]The most restrictive assumption is (roughly) that for any given bundle it is always possible to construct another bundle that is neither better nor worse.

a number with each bundle. A utility function for an individual must therefore provide a unique number for each bundle. Suppose we took the numbers provided by the function for any two bundles as a way to rank the bundles. Thus if the utility function provides a greater function value for bundle A than bundle B, then bundle A is preferred to bundle B. The important result is that for most preference relations there will be a utility function that ranks bundles in exactly the same way as the preference relation. Stated more formally, a utility function will exist that 'represents' the preferences of an individual where the numerical ranking provided by the function for any collection of bundles coincides exactly with the ranking of the bundles provided by the individual.

There are two important implications of this result. First, it is reasonable to think of wellbeing (or utility or welfare) as a number with larger numbers associated with a higher level of wellbeing. The wellbeing number associated with the bundle is just the value of the utility function at that bundle. Secondly, we have a simpler way of representing how a consumer ranks bundles. It is as if the consumer determined the relative rank of two bundles by evaluating a privately known utility function for each bundle and ranking according to the numerical value (meter reading) associated with the two bundles.

An example is useful. Suppose that an individual had preferences that were represented by a utility function of the form: $U^1(x) = x_1 + x_2 + x_3$ and further suppose that we are considering the ranking of three bundles: $x^A = (4, 2, 1)$, $x^B = (4, 1, 3)$ and $x^C = (3, 1, 3)$. The utility function values for the three bundles are: $U^1(x^A) = 7$, $U^1(x^B) = 8$ and $U^1(x^C) = 7$. We conclude that this individual prefers bundle B to both bundle A and bundle C and that he is indifferent between bundles A and C (they tie). Note that the utility numbers (7 and 8) are directly comparable because they are generated by the same utility function and are thus in the same units.

A final detail is that the utility function that represents the preferences of a consumer is not unique. That is, from any utility function that represents the preferences of a consumer, it is always possible to construct yet another utility function that does the job just as well. For example, above we used the example $U^1\ (x) = x_1 + x_2 + x_3$. Suppose we consider a new utility function which is just $U^1\ (x)$ multiplied by 2. That is: $U^4\ (x) = 2U^1\ (x) = 2x_1 + 2x_2 + 2x_3$. The new utility numbers for our bundles *A*, *B* and *C* are: $U^4\ (x^A) = 14$, $U^4\ (x^B) = 16$ and $U^4\ (x^C) = 14$. Again, the individual is found to be indifferent between bundles *A* and *C* and to prefer *B* to each of the others.

It is useful to examine briefly why the rankings were invariant to a rescaling (in this case, multiplying by 2) of the original utility function. The answer lies in noting that we are always comparing pairs of bundles and all that we need to know is which bundle is better. We don't need to know how much better one bundle is than the other. When the utility function was multiplied by 2 above, the relative positions of any two bundles was not changed. What did change was the irrelevant and arbitrary feature from the point of view of economic theory of how much better one bundle may be.

Implications for Valuation: A First Look

The foregoing discussion has suggested that thinking of the wellbeing of an individual as a numerical value determined by a utility function is consistent with an individual having a preference relation which allows him to rank bundles of goods and services in terms of desirability. It may be helpful to reconsider some of the earlier discussion on measures of value in light of the results about preference representation. Recall that in Section 2 we noted four popular uses of the term value. The first was in terms of (psychic) wellbeing units, the second involved the dollar equivalent of these units, the third involved expenditures and the fourth required trade-off and compensation for a fixed level of wellbeing. The utility function approach helps to clarify why the first two notions of value have limited appeal.

In the first place, we note that if two individuals have different preferences then their wellbeing units will tend to differ. The different preferences will, by definition, be represented by different utility functions and the units of these functions will differ. As such, the individual utility readings cannot be used to compare the wellbeing of two individuals. Secondly, we note that we will never in general be able to deduce the utility function of a consumer. An infinite number of utility functions will represent a given set of preferences and none of these functions need be in the same units. Recall that we can always create another representative utility function by multiplying by a positive number. It follows that any attempt to measure units or levels of wellbeing (as in a) will provide purely arbitrary results. As well, any attempt to associate dollar values with arbitrary wellbeing levels (as in b) must also be arbitrary. It is necessary to give up the notion of measuring wellbeing levels or changes and the value of wellbeing or wellbeing changes.

Summary of Preferences

It is useful to summarize the important points in the foregoing discussion of preferences.

1. It is assumed that an individual is able to rank any pair of bundles in terms of desirability or preference. Further, an individual is able to determine the best and the worst bundle in a given collection of bundles of goods and services. The ranking of bundles of goods and services is independent of whether these bundles are affordable.
2. The process by which an individual ranks bundles can be thought of as ranking bundles according to the numerical value each is given by a

representative utility function. The utility function summarizes all the relevant information about the preferences of the individual.

3. The utility function of an individual is not unique. Any representative utility function can be used to construct another that is just as good. It may help to think here of the relationship between Fahrenheit and Celsius temperature scales. Each scale will provide the same temperature rank of any two substances in terms of its 'own' measure of degrees. Any given substance will have a different temperature on the two scales (except for a substance at -40 degrees) and thus it is not generally the case that the same temperature on the two scales refers to the same substance.

The discussion of preferences allows us to dispense with the first two measures of value (a and b). In order to explain and evaluate the other measures it is necessary to introduce more aspects of the economic theory of value. We next consider how individual choices are constrained.

Constraints faced by Consumers

In the foregoing sections we discussed the nature of the consumer's preference relation and how it could be represented by a utility function. Preferences were not restricted to only those bundles that the individual could afford. In this section we examine how constraints limit the goods and services that are available to the consumer. We will consider two types of constraints: the budget constraint and availability or rationing constraints.

The budget constraint operates in a fashion familiar to everyone. In its simplest form the budget constraint requires that the dollar value of the sum of all expenditures on goods and services made by an individual cannot exceed total income plus allowed borrowing. The effect of the budget constraint is to define a subgroup of bundles of goods and services that are affordable. This subgroup is referred to as the 'budget set' of the consumer. An individual with a larger income will have a budget set that contains more bundles than those in the budget set of a lower income individual. Similarly, if one or more prices increase then fewer bundles are affordable and the budget set of the individual must shrink.

A second constraint sometimes arises from rationing. This is not a financial constraint. Rather, it usually appears as a constraint on the maximum number of units of a good or service that a consumer is allowed to purchase regardless of the price of the good or the income of the consumer. Examples of these types of constraints sometimes appear when a good goes on sale at a given price but there is a limit of, say, two units per customer. As with the budget constraint the rationing constraint serves to reduce the bundles that are available to the consumer.

In sum, budget and other constraints define the bundles of goods and services that are both affordable and available to an individual. Given that a consumer's preference relation applies to all combinations of goods and services, it can be used to rank the bundles in this smaller group of bundles that are both affordable and available.

The Rationality Postulate in Economics

A cornerstone of economic models of the consumer and therefore of the economic theory of value is the rationality postulate. For our purposes, the postulate can be stated as follows: the consumer will always choose the best affordable and available bundle. The operation of the rationality postulate can be visualized in the following way. First, the budget and rationing constraints define the group of bundles that is both affordable and available. Second, the consumer then uses his preference relation to rank all of the affordable and available bundles. In the case of no ties, one bundle will emerge as the best (highest ranked or most desirable). This is the best affordable bundle and the one that the rationality postulate requires the consumer to choose. Note that if the consumer has a utility function that represents his preferences then the best affordable bundle will yield the greatest (or maximum) value of the utility function when it is evaluated at all feasible (affordable and available) bundles.

In a sense, the rationality postulate is the glue that joins the components of the consumer model. Without the postulate we have a passive individual described by his preferences over all goods and services and by a group of bundles that he can afford and which are available to him. With the rationality postulate the consumer becomes active to the extent that he chooses according to his preferences the best affordable and available bundle.

Defining the Equilibrium State of a Consumer

The previous section defined the bundle that the consumer would purchase. Unless there is some change in the preferences or the budget of the consumer, the best bundle will remain unchanged and the consumer will have no desire to change his decision by choosing a different bundle. The consumer is thus in a state of rest or an equilibrium position. The 'visible' part of this equilibrium is that the consumer is observed to purchase given amounts of goods and services. These quantities are the components of the equilibrium bundle. Thus the economic theory of the consumer explains observed behaviour of individuals in the marketplace as the equilibrium outcome of a process involving preferences, constraints and rational choice. This proposed explanation of 'why' individuals are observed to behave as they do in an economic setting is an important achievement of the theory of value.

The Equilibrium of the Consumer May Change

The previous section suggested that an important contribution of the theory was to explain 'why' certain behaviour is observed. Equally as important for theoretical understanding and practical applications of value theory is the way the equilibrium construction can be used to answer 'what if' type questions. As the name suggests, these questions arise when one is attempting to assess the sensitivity of the consumer equilibrium to changes in underlying conditions such as the budget. For example, the model can be used to determine the best or equilibrium response of the consumer to an increase in the price of one or more goods. There is much to learn from the answers of these sorts of questions. Indeed, economic values arise as the answers to 'what if' questions. Within the economics literature the posing and answering of 'what if' questions is often referred to as comparative static analysis. In what follows we will examine the effects on the consumer equilibrium of changes in prices, income and quantity constraints. These experiments set the stage for determining economic values.

Changes in Equilibrium Due to Price Changes

The comparative static analysis of the effects of a price change works in a straightforward way. As an illustration, we will examine the implications for equilibrium consumer behaviour of an increase in the price of one of the goods. To clarify the issues, it is helpful to return to our example where the three goods available are food, opera tickets and fishing trips. We will suppose that, at the current equilibrium, the consumer decides that his or her best decision is to buy 6 units of food, 2 opera tickets and 3 days fishing. We can use our previously developed notation to write this more compactly as: $x^A = (6, 2, 3)$. Bundle x^A is just the initial consumer equilibrium. The quantities (6, 2, 3) are sometimes referred to as the consumer demands.

Suppose that the price of food increases but that nothing else in the budget constraint or preferences of the consumer changes. It is not likely that the consumer equilibrium will continue to be bundle x^A. Indeed, if the consumer was spending all of his income to purchase x^A then he would no longer be able to afford this bundle after the price of food had increased. In any event, the group of bundles that are both affordable and available has shrunk as a result of the price increase. The consumer's money 'does not go as far as it used to'. Given that x^A is no longer affordable, the consumer is out of equilibrium. We assume that he will attempt to get back to a new equilibrium position. To this end, he must use his preference relation to determine a new best bundle out of the group that is now both affordable and available. The fact that a consumer attempts to regain equilibrium is just an implication of the rationality postulate. It would not be in the consumer's interest to spend any amount of time away from his best available or most desired consumption bundle.

To continue our example, suppose that the new best bundle is $x^B = (4, 2, 4)$. The new equilibrium has two fewer units of food, the same number of opera tickets and one more day fishing. All of these changes constitute the equilibrium response to a given change in the price of food with all other prices and income held fixed. Note that even though just the price of food changes, the best response on the part of the consumer was to change not only the quantity of food purchased but also the quantity of at least one other good. In this case, the number of fishing trips increases. Finally we observe that in this example as the price of food increased, other things being unchanged, the quantity of food purchased decreased. This is a direction of change that one might expect.[8]

[8]The general consumer model does not guarantee that quantity demanded will always decrease when price increases. At the same time, there is no undisputed empirical evidence that this has ever happened.

Graphing Price Induced Changes in Equilibrium: The Demand Curve

Demand curves arise in a variety of forms throughout economic theory. We are now in a position to illustrate how one type of demand curve arises. It is named the Marshallian demand curve of the consumer after Alfred Marshall, one of the principal contributors to economic analysis. The Marshallian demand curve provides a graphical summary of some of the changes that occur in a consumer's equilibrium in response to changes in the price of a good. This is best illustrated by returning to our example.

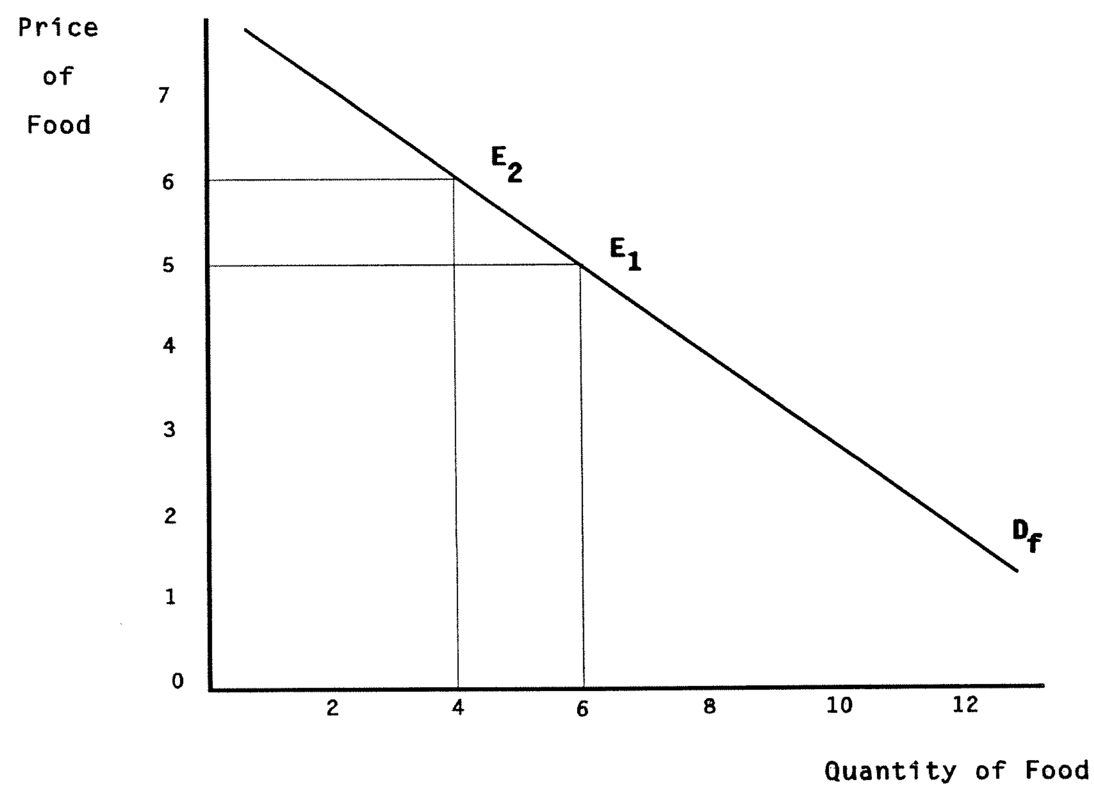
Recall that in the example the price of food increased and the consumer went from equilibrium at the bundle $x^A = (6, 2, 3)$ to a new equilibrium with the bundle $x^B = (4, 2, 4)$. We will suppose that the original price of food was $5 per unit and that it increased to $6 per unit. Further, we will fix our attention on what happens to the food component of the bundles as we move from equilibrium to equilibrium. Thus, when the price of food was $5 per unit the optimal quantity of food to purchase was 6 units. When the price increased to $6 per unit, the optimal quantity dropped to 4 units of food.

We now consider what a graph of this information would look like. We wish to construct a graph of the price of food against the quantity of food in the best bundle.[9] The result would be something like what is pictured in Figure 1. In Figure 1 the two points E_1 and E_2 coincide with the two consumer equilibria: at E_1 the price is $5 and the quantity is 6 whereas at E_2 the price is $6 and the quantity is 4.

Next, consider the downward sloping line D_f that joins E_1 and E_2 and extends on either side of these two points. This is the Marshallian demand curve for food. This curve traces out all values of the food component of the consumer equilibrium as the price of food changes and all other prices

[9]Recall that no other prices or income change as the price of food changes.

Figure 1: An Illustrative Demand Curve



and income remain fixed. It is the curve that would result if we repeated our price change experiment for all prices instead of just \$5 and \$6. As drawn, our curve is downward sloping. Whenever the equilibrium quantity demanded of a good declines as its price increases (with all other prices and income fixed) then the demand curve must be downward sloping. This inverse relationship is what is often popularly referred to as the 'law of demand'.

As a last point, we note that different individuals may have different demand curves for a good such as food even when they have the same incomes and face the same prices for all goods. This simply reflects a difference in the underlying preference relations of the individuals. Even with the same budgets they may identify a different equilibrium (best) bundle. It follows from the fact that the demand curves differ that individuals may respond with differing intensities to price changes. A small price change may do little to affect the purchasing patterns of one individual but may lead to large changes in quantity demanded by another. Economists have imported a measure called 'elasticity' from the physical sciences to quantify how quantity demanded responds to price changes. Suppose that quantity demanded changes by a certain percent in response to a given percent change in price. Elasticity is often measured as the (absolute value of the) ratio of the percent change in the quantity demanded to the percent change in price. In our example the quantity demanded declined by 33⅓% (from 6 to 4) in response to a 20% (from 5 to 6) increase in price. This leads to an elasticity measure of 5/3 calculated as the ratio: (33⅓/20). The greater is the elasticity measure, the more responsive is quantity demanded to price. One final feature of the elasticity measure is that it has no units (for example, price or quantity) associated with it. It is strictly the ratio of unitless percent changes. This property means that elasticity measures can be used to compare the responsiveness of different goods.

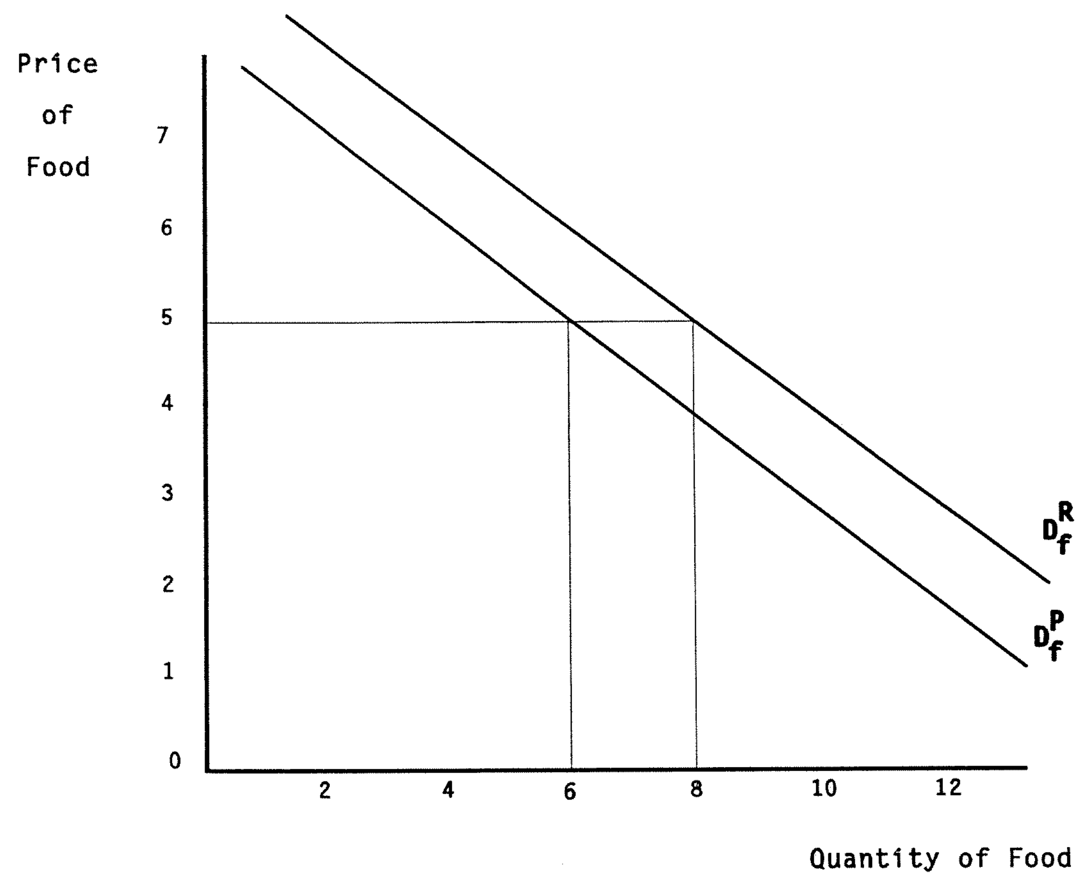
Changes in Equilibrium Due to Income Changes

The comparison of consumer equilibrium positions can be extended to the case where the income of the consumer changes with all prices held fixed. If income increases, for example, then the number of bundles that the consumer can afford increases. That is, the budget expands. It is reasonable to expect that the consumer will find that the best affordable bundle changes. As a result he will move to a new equilibrium position. We next use examples to illustrate the characteristics of this new equilibrium. The discussion that follows is quite important. An understanding of how income changes can by themselves affect the consumer equilibrium is important for understanding how income changes can be used for compensation in the valuation experiments that we will be considered later in this paper. These valuation experiments ultimately define economic values.

Suppose there are two individuals who are identical in every respect except that one individual has more income than the other. While there is no guarantee of this, we might find that the richer individual always purchases more food than the poorer individual. In the foregoing sentence we will use the word 'always' in the following more precise sense: regardless of the price of food (which is the same for both individuals), the rich person always demands (purchases) more food than the poor person. This has an immediate implication for the Marshallian demand curves of the two individuals. As pictured in Figure 2, the demand curve for the rich person D^R_f is 'outside' the demand curve for the poor person D^P_f. When the price is fixed at \$5 per unit for our example curves, the poor person consumes 6 units of food while the rich person, we suppose, consumes 8 units.

It is helpful to consider an alternative interpretation to the two demand curves in Figure 2. Suppose that instead of comparing two individuals who differed only by their incomes, we compared one individual before and after he received a gift of income. As such, we can think of the income gift as transforming the individual from poor to rich. The demand curve D^P_f would be the 'before' curve and the demand curve D^R_f would be the 'after' curve for the same individual. Thus, it would appear that an increase in income with all prices held fixed tends to shift the demand curve of the individual. In this case the demand curve shifted out as income increased since more was demanded at each price.

Figure 2: Demand Curves And Income Changes



As a final point, it should be recalled that the graphical discussion should not be allowed to obscure the fundamental equilibrium process that is generating the diagrams. Income changes and this leads to a change in the group of bundles that are both affordable and available. If, as we suppose, the new budget contains a new best bundle for the individual then the individual will change his consumption pattern. Alternatively stated, the individual will move to a new equilibrium position.

Changes in Equilibrium Due to Quantity Restrictions

It will sometimes be the case that the bundles available to a consumer are restricted by more than just affordability considerations. Rationing is an example. In times of crisis (such as wars) gasoline is sometimes rationed. Rationing has the effect of restricting the budget set of an individual to those bundles that are both affordable and permitted or available. The introduction of a rationing constraint with prices and income held fixed will tend to change the best bundle that is both affordable and available to the consumer. As such, a rationing constraint will tend to change the equilibrium of the consumer. As well, just as a price change in food or an income change could be expected to change also the consumption of goods other than food, rationing of one good can reasonably be expected to change the equilibrium purchases of all goods.

The Relationship of Wellbeing and Budget Size

In the previous sections we observed how changes in prices, income and availability affect the budget set of the individual and how these changes generally cause the individual to select a new equilibrium bundle. In some cases it is possible to deduce how the wellbeing of an individual changes in response to a budget change. In particular, if in a given setting (call it 'A') the budget of the individual contains at least all of the bundles affordable and available in another setting (call it 'B'), then the individual is no worse off in setting A than he was in setting B. Alternatively stated, the individual is no better off in setting B. The reasoning to establish this is as follows. In setting A the consumer could always, if necessary, throw away bundles to make his budget identical to the one in B. Thus, he can always guarantee that he is identical to B and hence no worse off than in B.

Measuring Values in Economics

Introduction

Recall that at the outset of this paper the decision was made to provide more background on economic decision making and not to start the analysis of economic values with a discussion of demand curves. This decision has led to a considerable lengthening of this paper. On the other hand, there are two benefits. First, the process of describing the economic model and deriving the demand curve provides a sense of how economic models work and how economists reason. Secondly, a foundation has been laid for understanding how values emerge from economic models. It is now time to address the economic valuation issues and problems that were raised in Section 2. The economic model of the consumer yields several candidate approaches to valuation. Some are essentially uninteresting while others are quite subtle.

Two principal aspects of the economic theory of value involve the individual's attainment of an equilibrium position and the study of how that equilibrium may change as the result of changes in prices, income and quantity restrictions. By carefully adding constraints or restrictions into the consumer model we can simulate a variety of policy changes. For example, we can use the equilibrium model to simulate the effect on an individual of restricting access to or closing a recreational fishery. To illustrate this, we reconsider our familiar 3 good model involving food, opera tickets and fishing trips. In order to simulate the fishery closure we could, as a first approximation, restrict the budget of the consumer so that he is not allowed to purchase any fishing days.[10] Given that the individual had chosen a positive number of fishing days in the pre-constraint

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[10]In a more complete model we might model closure as follows: the individual is allowed to take the fishing trip but is not allowed to retain any fish. This approach allows for the possibility that there is value associated with aspects of the trip other than just catching fish. Landing fish may be seen as an enhancement to the quality of the trip.

setting, he will be forced to choose a new equilibrium once the constraint is imposed. The effect, then, of the fishery closure is to restrict the bundles that are affordable and available and thereby to force the individual to move from one equilibrium to another.

Most policy changes will result in a change in the consumer's equilibrium. The goal of valuation analysis is to find measures of value that can be used to compare these equilibrium positions. The measures must be in observable units that are comparable across individuals. As suggested in Section 2 measures of value based on comparisons of levels of wellbeing or the dollar equivalent of those levels are inappropriate. In this section, however, we will examine several measures of value that meet our requirements. In particular, we will examine four compensation based measures of value. These measures are routinely advanced by economists. Which measure of value is used depends upon whether a price or a quantity change is being valued. The choice also depends upon when the value is measured: before or after the price or quantity change.

A General Statement about Economic Values and Compensation

We have shown that economic values cannot meaningfully be measured using either the change in the level of wellbeing of a consumer between two equilibrium positions or as the dollar 'equivalent' of a wellbeing change. There will always be a problem with the arbitrariness of the units of measurement of any wellbeing change. The only situation where the units of measurement would not be a source of concern is if the level of wellbeing was the same in the two equilibria that were being compared. This leads to several important questions which we will attempt to answer. For example: Can we define a reasonable and unambiguous measure of economic value when the level of wellbeing is unchanged? If so, can we then obtain these 'values' as the answer to 'what if' type questions in the consumer model? Can these new values be compared across individuals? Can the values be estimated using statistical techniques? The answer to all of these questions is yes.

We will consider four types of economic values that arise when wellbeing changes are restricted to zero. Two of these measures are appropriate for valuing price changes and the other two apply to the valuation of changes in quantity restrictions. All of these values have their origins in the work of Sir John Hicks, one of the founders of modern economic analysis. All of the value measures represent the answers to 'what if' questions posed in the consumer model. In contrast to the single price and quantity experiments that we introduced earlier, these 'what if' questions have two components. A price or a quantity change is introduced into the model and, at the same time, the individual receives (or pays) income compensation to guarantee that wellbeing is not changed. While the budget constraint of the consumer shrinks in one direction (as a result, say, of a price increase) it simultaneously expands in another direction as the income compensation is paid. In one case that we consider, the amount of compensation is such that the consumer will move to a new equilibrium where he is no worse and no better off than he was at the original equilibrium. Value is measured as the amount of income compensation that is paid. This compensation can be thought of as the minimum amount of money necessary to convince the consumer to move 'freely' from one equilibrium to another. Since this value is measured in dollars, it can be compared across individuals.

In the sections that follow we will examine all four compensation-based measures of economic value. We will postpone discussion of the estimation of these values in applied work to a later paper.

Valuing a Price Change: Compensating Variation

Consider a policy (perhaps a per unit tax) that causes the price of one good to increase. Implementing this policy would shrink the budget set of the individual and, given that he consumed the good, make the consumer worse off. Imagine, alternatively, that at the same time as the price was increased, the consumer received a compensating transfer of income. This income transfer will be just large enough so that the consumer can reach a new equilibrium (with a higher price and higher income) which is no worse in terms of wellbeing than the original equilibrium. That is, the consumer would willingly switch between the two equilibria. The income transfer is called the compensating variation (CV) associated with the price increase.

The idea behind the CV measure is to determine the amount of money necessary to guarantee that the wellbeing of the individual is unchanged from its original level. It is not a measure of how much the individual is harmed as a result of the price increase. We have already dismissed the possibility of measuring the value of a utility change. Rather, CV is a measure of what it costs to keep the individual from being harmed. It is the money value of keeping wellbeing unchanged.

The smallest value that CV can take on when a price increases is $0. This would be the case for an individual who does not consume the good that has increased in price. There is no upper bound on the value of the CV measure for a price increase. As we will illustrate in a numerical example in Section 4, because of the nature of their preferences it may be extremely expensive to compensate some individuals for a price increase.

When the experiment involves a price decrease, the CV measure of value is the maximum amount of money that can be taken from the consumer so that he does not benefit from the price change. In this case, the smallest value of CV would be $0 when the good is not consumed. The largest value would be the total income of the consumer. If the price declines and the consumer does not purchase the good then no income needs to be removed. Alternatively, if the consumer benefits from the price decrease, the most that can be taken from him is his total income.

It is possible to illustrate the compensating variation measure diagrammatically. There is, however, a small caveat. Our discussion of economic theory and demand curves has not been sufficiently detailed to allow for an exact representation of CV. In order to present an exact diagram we would need to introduce more theory and different types of demand curves. Nonetheless, we can provide a very approximate indication of what the CV measure looks like as an area 'behind' the Marshallian demand curve. In Figure 3, the CV of a price drop from $5 to $4 per unit of food is given by the area A + B. Because of the very approximate nature of our graphical presentation, the area A + B is also the same as the CV corresponding to a price increase from $4 to $5. The area A + B will tend to be greater for an individual with more income as this person's demand curve would be located to the right of the one in Figure 3.

In summary, the compensating variation value applies to a situation where one price actually changes and income is either added to or subtracted from the individual so that the original level of wellbeing of the individual is maintained. The amount of income compensation is called the compensating variation and it is thought of as the "value" of the price change such that the individual is no worse or no better off. In order to facilitate comparison with other measures of value, a summary is presented in Table 1.

Valuing a Price Change: Equivalent Variation

We again consider an experiment where the price of one of the goods changes. More precisely, a price change is 'threatened'. Instead of making the consumer live with the higher price and a compensated income (as in the CV case) the consumer is provided with the following option. He can purchase a guarantee that the price will not increase for him. Of course, his income will decline by the amount that he pays for the guarantee.

We can picture the process working in the following way. Imagine that the consumer is in his initial equilibrium and is threatened with a price increase that would force him to a new equilibrium with a lower level of wellbeing. The consumer is then 'sold' successive decreases in the price of the good from the higher threatened level. While the price decreases may tend to increase the wellbeing of the consumer relative to the new equilibrium, the 'selling price' for the guaranteed lower price is set as high as possible so that no increase in wellbeing arises. That is, any gain that may arise from a lower price is offset by the reduction in income (and hence wellbeing) as the consumer pays for the price decrease. This process continues until the consumer buys enough price decreases to bring the price down to the initial level. The consumer finds that he is in a final equilibrium where he is just as badly off as if he had simply accepted the price increase and not attempted to buy any reductions. The total amount of money that the consumer paid to secure the return to the initial price is called the equivalent variation (EV).

The EV value of a price increase is bounded from above by the total income of the individual. He cannot pay any more than that to secure a price decrease. As well, it is bounded from below by $0. This is the case of someone who does not purchase the affected good. In the case of a price decrease, the EV amount must be paid to the consumer. There is no upper bound on this amount. The lower bound is $0 for someone who does not buy the good.

The EV experiment is not identical to the CV experiment. As summarized in Table 1, with the EV case, the consumer is forced to stay at the post-policy level of wellbeing. Also, the price that the consumer pays for the good in the EV case is ultimately unchanged. In general, the EV and CV values will differ. A judgment must be made as to where wellbeing is held fixed. This is more a question of experimental design that of economic 'correctness'.

Finally, it is again possible to present the EV measure diagrammatically. Because we have adopted an approximate approach, it is not possible to present a clear distinction between EV and CV in a diagram. In terms of Figure 3, the EV measure of a threatened price decrease from $5 to $4 is A + B. Similarly, this is approximately the same value as for a threatened price increase from

\$4 to \$5. Note that is our approximate setting, CV and EV are both the same. As the example in the next section will show, CV and EV will generally differ.

Valuing a Quantity Change: Compensating Surplus

The compensating surplus (CS) quantity experiment is conceptually very close to the compensating variation (CV) price experiment. In the CS case we suppose that an individual, currently in equilibrium, is faced with a new constraint on the maximum quantity that he can buy of one of the goods. For example, the consumer could be forced to accept 1 more unit of a good. Alternatively, such a constraint would make the consumer worse off if he were told that a ration had been set at one less unit. Rather than let the wellbeing of the consumer change from the initial level, the consumer receives or pays the smallest transfer of income so that, with the constraint in place, he can buy another bundle which is just as good as the initial bundle. The final equilibrium of the consumer will involve a constraint on one of the goods, an income change to compensate for the constraint and a level of wellbeing that is identical to the pre-policy level. The extra income that the consumer receives or pays is called the compensating surplus (CS). These results are summarized in Table 1.

It is somewhat more difficult to work out bounds on CS than it is for CV. At this point we note that CS is always bounded from below by \$0 but that it can be unbounded from above. As well, there is no connection between CV and CS. One is a price experiment and the other is a quantity experiment.

As with the price experiments, it is possible to provide an approximate diagrammatic representation of CS. In Figure 3, the area B + C + D approximately corresponds to the most that a consumer would pay for an increase in his ration of food from 6 to 8 units. Again in an approximate sense, the area B + C + D represents the amount of money that the consumer must receive in compensation for a ration reduction from 8 to 6 units of food.

As a final point, we could briefly reexamine the question: What is the value of the recreational fishery? One answer is just the CS associated with constraining the number of fishing days to zero. This value is in dollars for any one individual and the total value of the fishery would involve adding up the dollar CS amounts for all affected individuals. CS is the smallest amount of money compensation that would have to be paid to recreational fishermen in order to get them to 'freely' choose not to fish.

Valuing a Quantity Change: Equivalent Surplus

The equivalent surplus (ES) measure of value is the quantity counterpart of the equivalent variation (EV) measure of the value of a price change. We begin with an individual who would have to move to a new equilibrium if a quantity constraint were imposed. The consumer is offered the opportunity to buy a guarantee that the quantity constraint will not be imposed. The most that the individual would pay for this guarantee would be that amount of money such that, after the guarantee was secured, the consumer was no better off than if he simply accepted the quantity constraint. This amount of money is called the equivalent surplus (ES). In the final equilibrium the consumer will face no constraint but, because his income is reduced, he will have the same level of wellbeing as if the constraint had been imposed. In contrast to the compensating surplus case, wellbeing is fixed at the post-policy level and the threatened constraint is ultimately not imposed on purchases of the good.

As in the CS case, ES is bounded from below by \$0. Upper bounds are illustrated in the next section. There is no link between ES and EV values. ES values arise from quantity experiments and EV values arise from unrelated price experiments. We note that ES is also an answer to the question: What is the value of the recreational fishery? In general, the ES answer will differ from the CS answer. Economic theory provides no clear guidelines for choosing between the two.

The approximate graphical representation of EV for both quantity increases and decreases is given in Figure 3 as B + C + D. This area is the same as for CS and is only so because of the approximate nature of the analysis.

Some Final Comments and Cautions

As discussed in the foregoing sections, economic values correspond to highly structured experiments performed within the economic model of consumer behaviour. Depending on the experiment, different economic values arise. In particular, the appropriate economic value to measure depends upon whether a price or a quantity change is being contemplated and then upon where the level of wellbeing is pegged (at the pre- or post-experiment level). In contrast to our discussion, we have only been able to use diagrams to illustrate broad differences between the measures of value. It is important to emphasize that all four measures will, in general, yield different dollar values. It is particularly important to be clear about the fact that the 'variation' measures (CV and EV) correspond to price experiments and that 'surplus' measures (CS and ES) correspond to quantity experiments. It is generally entirely inappropriate to suggest that CV or EV measures can be useful compared to or substituted for CS and ES measures. They correspond to entirely different experiments and will generally bear no resemblance to each other. In other words, it is not appropriate to use price experiment values to determine the 'value' of a quantity policy (and vice versa).

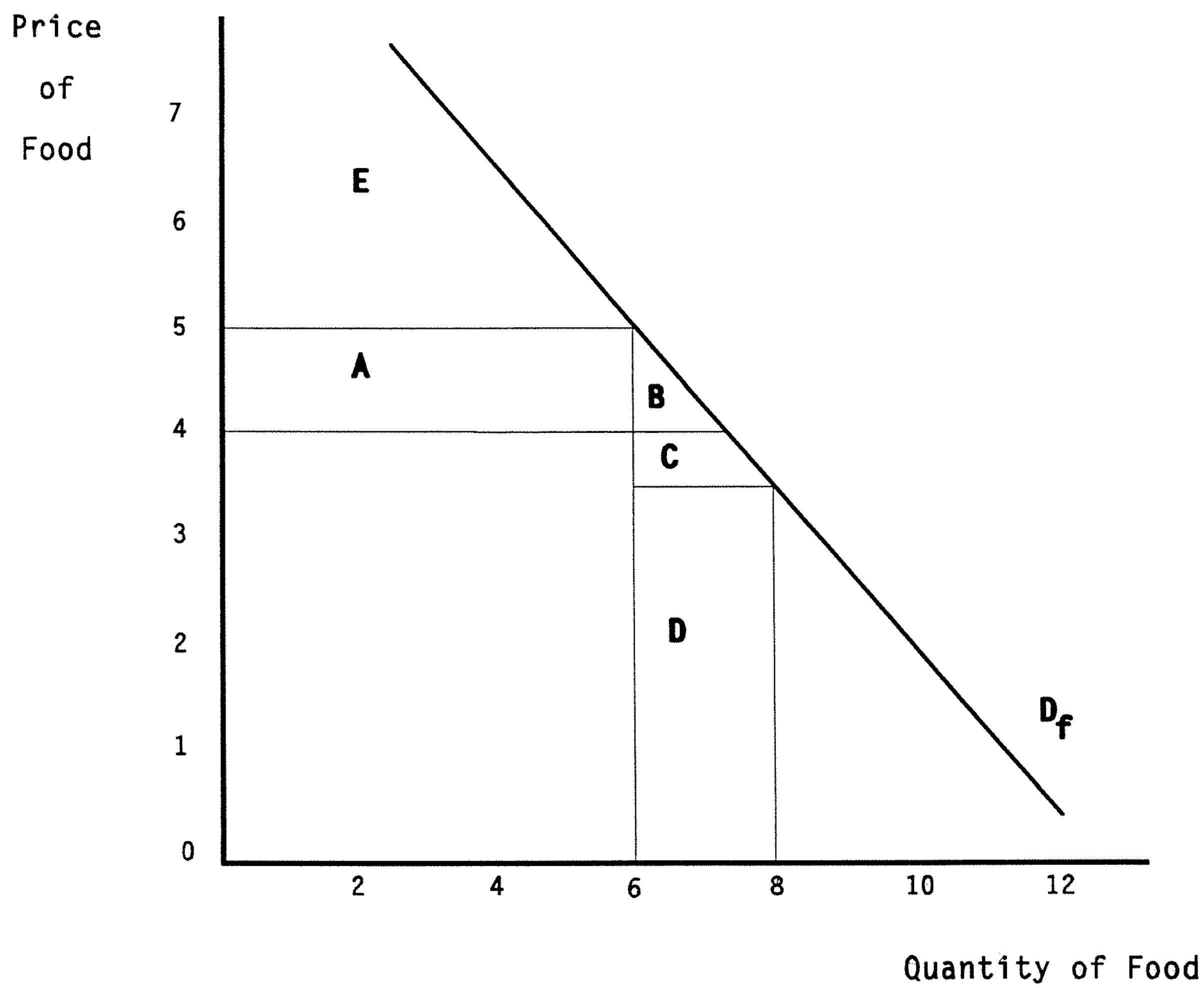
A last point concerns use of the popular phrase 'consumer's surplus'. This expression has not been used before in the paper. We raise it now in anticipation of any confusion that may arise when the contents of this paper are compared with other treatments of the topic. Recall that we suggested that CV and EV were approximately (but almost never exactly) equal to the area A + B in Figure 3. Many authors refer to the area A + B as the consumer's surplus associated with a decrease in price from $5 to $4. Strictly speaking, consumer's surplus will almost never correspond to a well-defined experiment within the consumer model. As such, it is difficult to give it any meaning as an economic value. Often, though, it will be approximately equal to CV or EV. The ease with which it can be graphed and calculated and its approximation property have kept the consumer's surplus term current in the economics literature.[11]

[11]Some authors refer to an area such as E+ A + B in Figure 3 as consumer's surplus. The only difference with our discussion is that E + A + B corresponds to a drop in price from an arbitrarily high price to $4.

TABLE 1: FOUR ECONOMIC VALUES

WHAT HAPPENS TO ...	NATURE OF THE EXPERIMENT			
	CV Compensating Variation	EV Equivalent Variation	CS Compensating Surplus	ES Equivalent Surplus
1. price of good	changed by design	change is threatened but ultimately not implemented	no change	no change
2. quantity of good	adjusts residually (not part of experimental design)	adjusts residually (not part of experimental design)	changed by design	change is threatened but not implemented
3. income level (net of compensation)	changed by design	changed by design	changed by design	changed by design
4. value	given the price change, CV is the amount of income compensation necessary to keep wellbeing fixed at the initial (pre-policy) level	price does not change but EV is the amount of income compensation needed to keep wellbeing fixed at the level that would have arisen if the price had changed	given the quantity change, CS is the amount of income compensation needed to keep wellbeing fixed at the initial (pre-policy) level	quantity ration is not changed but ES is the amount of income compensation needed to keep wellbeing fixed at the level that would have arisen is quantity had changed

Figure 3: Marshallian Demand and Approximate Economic Values



4. A Numerical Example

Introductory Comments

The example that is examined in this section is not necessary for an understanding of the economic values introduced earlier. Further, as an example, it shows nothing more than a special case that probably has more value for its numerical tractability than for its realism. With this understanding, however, the example may provide some extra insight into the economic modelling process where we move from a description of preferences and constraints, through the determination of the consumer equilibrium and, finally, to the compensation experiments that define economic values.

Definition of Preferences

For this example we will assume that there are two goods over which the consumer has preferences. These goods, the amounts of which we will represent generically as x_1 and x_2, may be thought of as representing recreation and food. The utility function representing the preferences of the consumer is defined by:

$$U\,(x_1, x_2) = x_1\, x_2 \tag{1}$$

That is, wellbeing is determined by the levels of the goods and their interaction (multiplication). As long as one good is consumed (i.e. is greater than 0) then more of the other good will always make the consumer better off because the product of the two quantities will be larger. Within the economics literature this is known as the Cobb-Douglas utility function and it is frequently used in examples.

Definition of the Budget Constraint

We will assume that the consumer receives income equal to \$I.[12] Further, we will suppose that prices for goods are given by $\$P_1$ and $\$P_2$. These prices are assumed to be fixed and beyond the control of the consumer. If the consumer buys x_1 units of recreation at price $\$P_1$ then the total cost of recreation goods is $P_1 x_1$. Similarly, the total cost of x_2 units of food at $\$P_2$ per unit is $P_2 x_2$. The total expenditures on the two goods may not exceed total available income.[13] We will assume that the consumer spends the entire budget on the two goods. This leads to the following equality linking expenditures and income:

$$P_1 x_1 + P_2 x_2 = \mathrm{I} \tag{2}$$

The sum of expenditures on the two goods just equal available income. This is called the budget equation or the budget constraint.

Rationality

Recall that the rationality postulate assumes that given her preferences and budget, an individual will make purchasing decisions that lead to the highest possible level of wellbeing. In our example this means that the consumer will buy that bundle of recreational services and food (x_1, x_2) so that the product $x_1 x_2$ is as large as possible and the budget is neither over- or underspent. The consumer will be in equilibrium with this bundle.

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[12]This income may come from working but we will not model the economics of the decision to work in this example.

[13]We will suppose that no borrowing is allowed.

Calculating the Consumer Equilibrium

The consumer allocation problem of finding the best bundle to purchase given the budget constraint is formally a problem in mathematical optimization. In what follows we will exploit some special features of our example and show how it can be solved is a more straightforward fashion.

Step 1: Note that the budget constraint can be rewritten:

$$P_2\,x_2 = \mathrm{I} - P_1\,x_1 \tag{3}$$

which states that what is 'left over' from income after spending $P_1\,x_1$ for recreation is just equal to what can be spent on good 2. Further, we can divide both sides by P_2 and arrive at:

$$x_2 = \frac{I - P_1 x_1}{P_2} \tag{4}$$

Equation 4 states that the number of units of good 2 (food) that you can buy is equal the total money left over for purchasing good 2 divided by the price per unit of good 2.

An important feature to note in Equation 4 is that since I, P_1 and P_2 are given, once x_1 is chosen, then x_2 can be immediately calculated. This is the nature of the constraint. The consumer can't choose both x_1 and x_2 freely. Once one is chosen, the other is immediately determined. Otherwise stated, any decision to choose x_1 immediately implies a 'chosen' value for x_2.

Step 2: The second step is to substitute Equation 4 into the utility function in Equation 1. Stated more carefully, we substitute for x_2 in Equation 1 the value that x_2 will take on whenever x_1 is chosen and the budget constraint is satisfied. Thus,

$$U = x_1\,\frac{(I - P_1 x_1)}{P_2} \tag{5}$$

Equation 5 provides the consumer's level of utility or wellbeing as it changes with good 1 with the understanding that good 2 is being chosen 'so to speak' residually to satisfy the budget constraint.

It is possible to 'complete the square' in Equation 5 and thereby provide an identical but different-looking expression for utility:

$$U = \frac{-P_1}{P_2}\left(x_1 - \frac{I}{2P_1}\right)^2 + \frac{I^2}{4P_1P_2} \tag{6}$$

Equations 6 and 5 are identical in the sense that expanding the right hand side of equation 6 leads to the same expression as the right hand side of Equation 5. Equation 6 shows the level of utility that arises for a given choice of x_1 given I, P_1 and P_2.

<u>Step 3</u>: We now consider the best choice of x_1 for the consumer so that utility, subject to the budget constraint, is as large as possible. The best choice of x_1 is that value of x_1 that makes the right hand side of Equation 6 as large as possible. Note, however that the first term on the right hand side of Equation 6 is always less than or equal to zero because it represents the product of a negative number $\left(\frac{-P_1}{P_2}\right)$ and a square $\left(x_1 - \frac{I}{2P_1}\right)^2$. Note as well that the second term does not depend on the choice of x_1. Thus, utility is made as large as possible when x_1 is chosen to make the first term as small a negative value as possible. We can always make the first term 0 by choosing:

$$x_1^* = \frac{I}{2P_1} \tag{7}$$

This is the optimal purchasing decision for good 1. Alternatively, this is the 'demand' for good 1. The star is put on x_1 to indicate that it is an 'optimal' decision.

The demand equation in equation 7 can be related to the demand curves introduced in Section 3 and illustrated in Figure 1. A demand curve arises when, for a given value of I, x_1 from Equation 7 is plotted against P_1. The reader will verify that there is an inverse or negatively sloped relationship between x_1* and P_1. Further, this curve will shift outwards as I is fixed at higher values. Finally, by substituting equation 7 into equation 4 the demand curve for good 2 is found to be:

$$x_2^* = \frac{I}{2P_2} \tag{8}$$

Maximum Level of Utility

All of the compensation measures include the maximum level of utility that can been achieved subject to constraint. When there are no constraints on the choice of x_1 and x_2 except for the budget then the maximum level of utility that the individual will achieve is:

$$U^* = \frac{I^2}{4P_1P_2} \tag{9}$$

This can be seen from equation 6. Where, when x_1 is optimally chosen, the first term is always equal to zero.

Suppose that, instead of being able to choose x_1 freely, its value is rationed at some level $\overline{x_1}$. The maximum utility that the individual will obtain is:

$$\overline{U} = \overline{x_1}\,\frac{\left(I - P_1\overline{x_1}\right)}{P_2} \tag{10}$$

An individual can never benefit from an extra constraint being imposed. Thus, it will always be the case that $\overline{U} \leq U^*$. The best that can happen for the consumer is for the rationed quantity $\overline{x_1}$ to coincide with the freely chosen demand. This will usually not be the case if someone other than the individual is setting $\overline{x_1}$.

Compensation Measures

1. Compensating Variation:

The CV measure of value for a change in the price of good 1 is implicitly defined by the following equation:

$$\frac{(I - CV)^2}{4(P_1 + t)\, P_2} = \frac{I^2}{4P_1P_2} \tag{11}$$

where t (which can be positive or negative) is the change in P_1. This can be rewritten:

$$CV = I - I\left(\frac{P_1 + t}{P_1}\right)^{1/2} \tag{12}$$

Equation 11 can be interpreted in the following way: CV must be set so that with a new price for good 1 equal to $P_1 + t$ and income compensated to I-CV, the level of utility of the individual (left hand side) is equal to the level of utility that would have arisen if the price had not changed and no compensation was paid (right hand side). From equation 12 we see that whenever $t < 0$ (i.e. price decreases) the value of CV is positive. As well, the greater is income, the greater must be the compensation when price increases. It is important to note as well that, in absolute value, CV may easily exceed the income level of the consumer. All we need is for the fraction $\left(\frac{P_1 + t}{P_1}\right)^{1/2}$ to exceed 2 and this happens, for example, when $P_1 = 1$ and $t = 4$.

2. Equivalent Variation:

The EV measure of value for a change in the price of the first good is given by:

$$\frac{(I + EV)^2}{4P_1P_2} = \frac{I^2}{4(P_1 + t)\,P_2} \tag{13}$$

where, again, t is the change in P_1. This can be rewritten:

$$EV = I\left(\frac{P_1}{P_1 + t}\right)^{1/2} - I \tag{14}$$

Details of the interpretation of EV are left to the reader but it is important to note that EV and CV are different.

3. Compensating Surplus

We use equation 10 to calculate the CS measure because it incorporates both the rationing and budget constraints in the calculation of the utility level. CS is defined implicitly by:

$$(\overline{x}_1 + T)\,\frac{(I - CS - p_1\overline{x}_1)}{P_2} = \overline{x}_1\,\frac{(I - P_1\overline{x}_1)}{P_2} \tag{15}$$

where T (positive or negative) is the change in the ration of good 1. It is further possible to solve for CS as:

$$CS = \frac{T}{\overline{x}_1 + T} (I - P_1\overline{x}_1) \tag{16}$$

From equation 16 it follows that CS is positive for increase in the ration ($T > 0$) and this means that removing CS dollars from the consumer will just compensate her for the increase in the ration of x_1. Similarly CS is negative for $T < 0$.

4. Equivalent Surplus

The equivalent surplus measure, ES, is implicitly defined by:

$$\overline{x}_1 \frac{(I + ES - P_1\overline{x}_1)}{P_2} = \frac{(\overline{x}_1 + T)\ (I - P_1\overline{x}_1)}{P_2} \tag{17}$$

Where T (positive or negative) is the change in the rationed quantity of good 1. An explicit expression for ES, obtained by solving equation 17 for ES is:

$$ES = \frac{T}{\overline{x}_1} (I - P_1\overline{x}_1) \tag{18}$$

Equation 18 should be interpreted in the following way. Recall that in the equivalent surplus experiment, utility is held fixed at the post experiment level. Thus, if T is positive and individuals have therefore been given T extra units of good 1, then their ex post utility will be higher than the ex ante level. ES is the amount of income compensation that needs to be given to the pre policy level to make sure that ex ante compensated utility equals the post policy level.

Some Numerical Examples

We close this section with some numerical examples. These examples provide another indication of how the compensation value measures can differ. The values of the price, income and ration parameters are given on Table 2. The price experiment is based on a decrease in the price of good 1 from \$1 to \$.5. In the rationing setting we assume that x_1 is initially rationed at 1 unit and this ration is increased to 2. The results are summarized in Table 2.

The results for the price experiment suggest that a individual will give up \$.59 to obtain a price decrease of \$.5 (CV = .59). Alternatively, this individual needs to be given \$.83 in compensation to be just as well off as if price had declined by \$.5.

The results for the quantity experiment suggest that an individual will give up, at most, \$.50 to receive a gift of 1 unit of good 1. Alternatively, the same individual needs to receive compensation of \$1 to be just as well off as if she had been given a true gift of 1 unit of x_1.

As a final point, we stress that price and quantity experiments to determine economic compensation values are conceptually different and not comparable. The nature of the numerical example allows us to shed more light on this assertion. Note that from the demand curve of equation 7, when I = 2 and $P_1 = 1$, $x_1^* = 1$. As well, when I = 2 and $p_1 = .5$, $x_1^* = 2$. From this, one might be tempted to conclude that the price experiment where price drops by .5 is equivalent to the quantity rationing experiment where quantity increases by 1. This argument is flawed. We can see from Table 2 that none of the price and quantity values coincide. Indeed, it is easy to construct examples where the difference is much more exaggerated. The reason why all the experiments differ is that they all affect the consumer's budget set in a different way and there will generally be a different amount of compensation required for each of these different experiments. When examining the results of applied economic studies where values have been estimated it is very important to verify that the appropriate value had been estimated form performing the necessary experiment. For example, the economic value of changing a rationed constraint (for example, a bag limit) cannot be obtained as the result of a price experiment.

TABLE 2: NUMERICAL EXAMPLES OF COMPENSATION MEASURES*

EXPERIMENT	CV	EV	CS	ES
1. Decrease P_1 by \$.5 from \$1 to \$.5 (t =-.5)	.59	.83	--	--
2. Increase $\overline{x_1}$ by 1 from 1 to 2 (T = 1)	--	--	.50	1.0

* <u>Initial Conditions</u>

$P_1 = P_2 = 1$

I = 2

$x_1^* = 1; \quad \overline{x_1} = 1$

$x_2^* = 1;$

$U^* = 1; \quad \overline{U} = 1$